An Efficient Double Auction Mechanism for Job Allocation

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Job Supply and Demand Market

General Manager
Quadro Earthmoving Pty Ltd
Geraldton, Gascoyne & Midwest
$150,000 - $200,000+
CEO & General Management > General/Business Unit Manager
- Geraldton Based • Relocation Available
- Overseeing Projects Within the Haulage, Mining & Civil
- Performance Based Bonus Offered
Quadro Earthmoving is seeking a General Manager to join their team.

Group General Manager
Caledonia Group Pty Ltd
Perth > CBD, Inner & Western Suburbs
CEO & General Management > General/Business Unit Manager
- Key strategic appointment
- Drive growth and profitability
- Renowned leader in our field
Drive the execution of business strategy and be part of an exciting team at Caledonia, an industry leading integrated services provider.

COO
Hayes Executive
Blue Mountains & Central West
CEO & General Management > COO & MD
Regionally based NFP seeking an Executive General Manager Operations

Roughly 93% of companies are searching for candidates on LinkedIn
Market Based Models

Two categories of market based models:

- Commodities Market Model
- Auction Model
  - one-sided auctions
  - two-sided auctions (double auctions)

The advantage of using auctions for resource allocation is that they require little global information, have decentralized structure and are easy to implement.
Auction Model on Job Allocation

If the existing auction mechanism is directly applied to solve the problem, it may **not achieve expected results**.
General Double Auction Model

- **Time First:** When the buyer and the seller have the same bid price, the buyer who bids earlier will prioritize transactions.
- **Price First:** The buyer who bids higher will prioritize transactions. In contrast, the seller who asks lower price will prioritize transactions.
- **Transaction Price:** The transaction price is generally the average of bidding and asking price. The premise of the transaction is that the seller's asking price is lower than the buyer's bid.
Greedy Double Auction Mechanism (GDA)

• The auctioned goods come from various positions offered by different companies are heterogeneous
• There is a one-to-one correspondence between the bidder and the auction goods
• Third-party platforms have access to all product (or job) as well as bid information, however, this information is private to both parties
Winner Decision Problem

\[\text{Max} \sum_{1}^{\mid X_{\text{win}} \mid} X_{ij}^k \left\{ [p_{ij} - \frac{p_{ij} - b_{ij}^k}{2}] + \left[ \frac{p_{ij} - b_{ij}^k}{2} - b_{ij}^k \right] \right\}\]

\[X_{ij}^k \in \{0, 1\}\]

\(X_{\text{win}}\) represents the solution which maximizes the social welfare in auction and meets the constrains.
GDA : Find Valid Bids

Algorithm 1: Valid Bids

Input: $J, S, B$
Output: $J_{potential}, J_{value}$

1 for $t = 1 \rightarrow |B|$ do
   2 if $B_t$ is marked with $i, j$ then
      3 Put $job_{ij}$ into $J_{potential}$
   4 end
   5 end
6 for $t = 1 \rightarrow |J_{potential}|$ do
   7 Read $J_t$ get $i, j$
   8 end
9 $index \leftarrow 0; count \leftarrow 0$
10 for $s = 1 \rightarrow |B|$ do
11 Search $b$ marked with $i, j$
12 if $b_{ij}^k \leq p_{ij}$ then
13 count=count+1;
14 Put $b_{ij}^k$ into $bid_{index}$
15 end
16 end
17 if $count > 0$ then
18 Put $job_{ij}$ into $J_{value}$
19 end

- Statistic the jobs we are competing, exclude jobs that are not competed, and put them into the set $J_{potential}$
- Compare the bid of company $p_{ij}$. If $p_{ij} \geq b_{ij}^k$ the transaction can be completed
GDA: D-value Sort

Algorithm 2: D-value Sort

Input: $J_{value}, Bid, S, J$
Output: NewBids

1. for $s = 0 \to |Bid| - 1$ do
2. \hspace{1em} Read $bid[s]'s, j, kindex D^{k}_{ij} = p_{ij} - b^{k}_{ij}$
3. end
4. Sort all $D^{k}_{ij}$ in $Bid$ while max put forward
5. Put result into NewBids

- Sort valid bids according to their difference between the value of position
GDA : Greedy Allocation

Based on the greedy mechanism, we give priority to the transaction where quotation and the bidding with large price difference, until all the positions are distributed.
The time complexity of the greedy double auction (GDA) mechanism is $O(m*n + 2n^2)$.
Individual Rational

The greedy double auction (GDA) mechanism is individual rational.

Proof: In algorithmic design,

- if a candidate lies about his true desire to apply for a job, he is unable to compete for a position. At this point, $p_i^S = 0, U^S = 0$.

- When the position $job_{ij}$ has only one valid bidder, if and only if $p_{ij} \geq b_{ij}^k$, then $U^S = \frac{(p_{ij} + b_{ij}^k)}{2} - b_{ij}^k$. Therefore there must be $U^S \geq 0$.

- When the position $job_{ij}$ has more than one effective bidder, there must be a lowest bidder. If the applicant $s_k$ has lowest bid, then $U^S \geq 0$. If the bid is not the lowest, then $U^S = 0$. 
Budget-balanced

Greedy double auction (GDA) mechanism is budget-balanced.

**Proof**: the transaction price between the buyer and the seller is the average of the buyer's bid and the seller's asking price.

\[
\sum_{job_{i,j} \in J_w} p_{i,j} - \sum_{s_j \in S_w} p^S_{i,j} = \sum_{s_j \in S_w} p_{i,j} - p^S_{\sigma(i,j)} = 0
\]
Greedy double auction (GDA) mechanism is **truthful**.

**Proof**: 

- the bid $b_{ij}^k$ lower than true bid:

  Job seeker wins bid, his new utility is $U_{S'}$. $U = (b + p)/2 - b$. Due to $b_{ij}^k \leq b_{ij}^k$, so $U_{S'} \leq U^S$. In this case, the false bid leads to a lower utility.

- the bid $b_{ij}^k$ higher than true bid:

  1). job seekers fails the competition, $U_{S'} = 0$.

  2). job seeker wins the bid, due to the utility function: $U = \frac{b + p}{2} - b = (p - b)/2$. The price of the position remains unchanged with the raise of bid $b$ provided by the seeker. As a result, job seekers’ utility declines.
DA & GDA Results on Social Welfare

- When the number of jobs remain unchanged, the increase in the number of job seekers is conducive to the increase of social welfare.
- The welfare will be reduced after reaching a certain value.
- GDA has higher social welfare than DA under the same conditions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
<th>Range</th>
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<tr>
<td>m</td>
<td>the number of positions</td>
<td>50</td>
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<tr>
<td>n</td>
<td>the number of job seekers</td>
<td>[20, 100]</td>
</tr>
<tr>
<td>A</td>
<td>the value of positions</td>
<td>[4500, 7500]</td>
</tr>
<tr>
<td>B</td>
<td>the bid of seekers for positions</td>
<td>[4000, 8000]</td>
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<tr>
<td>d</td>
<td>step length</td>
<td>10</td>
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</tbody>
</table>
Job matching rate

- With the increase of job seekers, the matching rate of DA and GDA has both increased.
- The matching rate of GDA is higher when there are fewer people in the early stage, but the matching rate of DA is higher than that of GDA when there are more people in the later stage.
The average utility of GDA is higher than that of DA.

With the increasing number of job seekers, the average seeker's utility of DA and GDA are basically stable and occasionally fluctuate.
Conclusion

- GDA has higher social welfare than DA under the same conditions.
- The matching rate of GDA is higher when there are fewer people in the early stage, but the matching rate of DA is higher than that of GDA when there are more people in the later stage.
- The average utility of GDA is higher than that of DA.

The mechanism has effectively improved social welfare and the average utility of job seekers.